Test 2 - MTH-1400 Online

Dr. Adam Graham-Squire, Summer 2018

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones, computers, notes and textbooks are not allowed on this test. Calculators are not allowed on the first ??? questions of the test. Calculators are allowed on the last ??? questions, however you should still show all of your work. You will initially receive the entire test, and you will NOT be allowed a calculator. Once you have finished everything you can without a calculator, you should turn in the first part of the test (the first ??? questions) to the proctor. The proctor can then give you your calculator and you can finish the remaining questions. You are not allowed to go back to the No Calculator portion once you have been given your calculator.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. If you need it, the law of cosines is $c^2 = a^2 + b^2 - 2ab\cos(C)$.
8. Make sure you sign the pledge.
9. Number of questions = 11. Total Points = 60.

1. (5 points) (a) Convert the angle $\frac{4\pi}{3}$ from radians to degrees.

(b) Find a *negative* angle (in degrees) that is coterminal with the angle $\frac{4\pi}{3}$.

(c) Find a *positive* angle (in radians) that is coterminal with $\frac{4\pi}{3}$ (Note: I am looking for some angle *other* than $\frac{4\pi}{3}$).

2 (a) $\frac{4\cancel{\pi}}{\cancel{3}} \cdot \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}} = 240^\circ$

1.5 (b) $240 - 360^\circ = \boxed{-120^\circ}$

1.5 (c) $\frac{4\pi}{3} + 2\pi = \frac{4\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{10\pi}{3}}$

2. (10 points) Find the following. You can use the unit circle on the next page to help you calculate the given trigonometric values, if you want, but filling in the unit circle itself will get you no points. If an expression is undefined, write DNE and briefly explain why it does not exist.

(a) $\sin\left(\frac{\pi}{6}\right)$= $\frac{1}{2}$

(b) $\tan\left(\frac{3\pi}{4}\right)$= -1

(c) $\sec\left(\frac{\pi}{2}\right)$= $\frac{1}{\cos(\frac{\pi}{2})} = \frac{1}{0} = \boxed{DNE}$

(d) $\cot\left(\frac{7\pi}{2}\right)$= 0

(e) $\cos\left(\frac{-\pi}{3}\right)$= $\frac{1}{2}$

(f) $\csc\left(\frac{4\pi}{3}\right)$= $\frac{-2\sqrt{3}}{3}$ or $\frac{2}{\sqrt{3}}$

(g) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$= $\frac{\pi}{3}$ or $60°$

(h) $\tan^{-1}(-1)$= $\frac{-\pi}{4}$ or $-45°$

(i) $\arccos(\sqrt{3})$= $\boxed{DNE}$ b/c domain of arccos is $[-1,1]$ and $\sqrt{3} >$

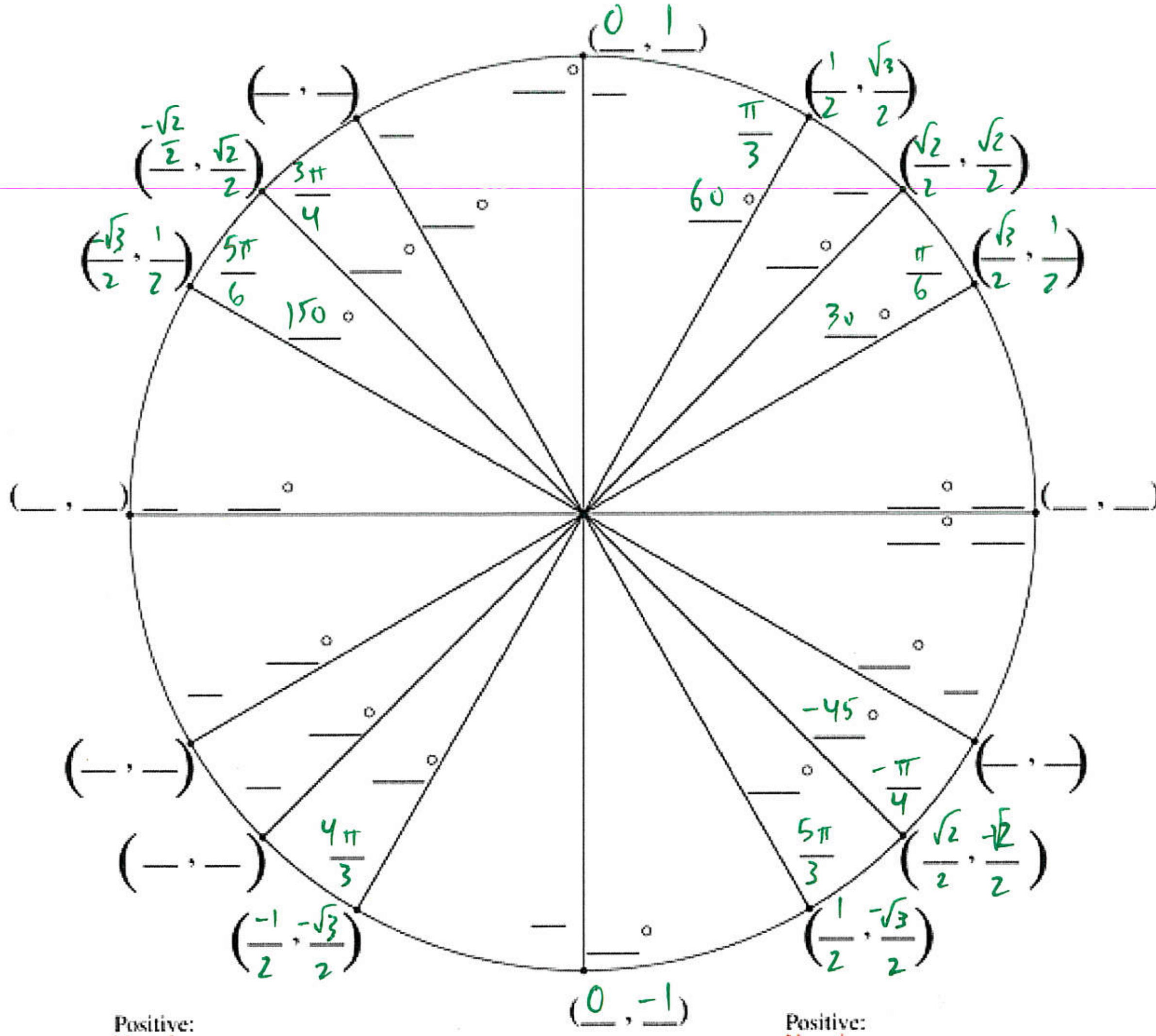
(j) $\sin(-930°)$= $1/2$

(d) $\cot\left(\frac{3\pi}{2}\right) = \frac{\cos(\frac{3\pi}{2})}{\sin(\frac{3\pi}{2})} = \frac{0}{-1} = 0$

(f) $\frac{1}{\sin(\frac{4\pi}{3})} = \frac{1}{(\frac{-\sqrt{3}}{2})} = \frac{-2}{\sqrt{3}}$ or $\frac{-2\sqrt{3}}{3}$

(j) $-930 + 720 = -210$

$-210 + 360 = 150°$ $\quad \sin(150) = \sin(30) = \frac{1}{2}$



Positive:
Negative:

Positive:
Negative:

3. (5 points) (a) Starting with the identity $\sin^2\theta + \cos^2\theta = 1$, divide both sides of the equation by $\cos^2\theta$ to find the trigonometric identity relating the values of $\tan\theta$ and $\sec\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

2

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$$\tan^2\theta + 1 = \sec^2\theta$$

0.5 if have $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ and then goes wrong.

(b) If $\sin\theta = \frac{2}{3}$, use trigonometric identities to find the value of (i) $\cos\theta$, (ii) $\sec\theta$, and (iii) $\tan\theta$.

3

Quad II

$$\left(\frac{2}{3}\right)^2 + \cos^2\theta = 1 \quad \Rightarrow \quad \cos^2\theta = 1 - \frac{4}{9}$$

$$\cos\theta = \pm\sqrt{\frac{5}{9}} = \boxed{-\frac{\sqrt{5}}{3} = \cos\theta}$$

5/9

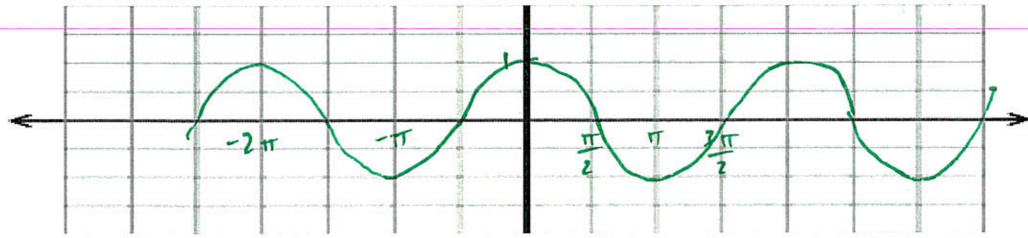
$$\boxed{\sec\theta} = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{-\sqrt{5}}{3}\right)} = \boxed{\frac{-3}{\sqrt{5}}} \text{ or } \frac{-3\sqrt{5}}{5}$$

$$(iii)\ \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{3}}{\frac{-\sqrt{5}}{3}} = \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{-\sqrt{5}} = \boxed{\frac{2}{-\sqrt{5}} = \tan\theta}$$

or $\frac{-2\sqrt{5}}{5}$

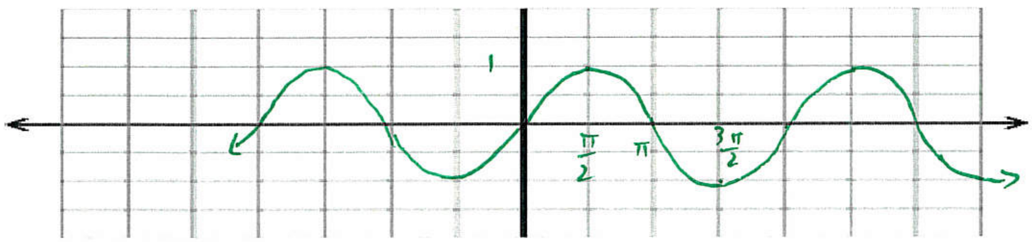
4. (5 points) (a) On the graphs below, sketch the graph for $y = \cos x$ and $y = \sin x$ (make sure to label some x-intercepts on each graph).

$y = \cos x$:



3

$y = \sin x$:



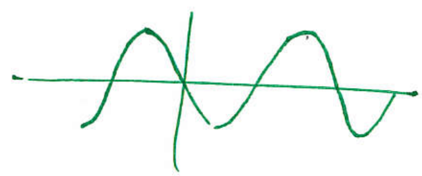
(b) Use the graphs to explain (in words) why the trigonometric identity

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin(x)$$

is true (if you cannot use the graphs above, you can also try to explain it using the unit circle, though that may be more difficult).

2

$\cos(x + \frac{\pi}{2})$ shifts the cosine graph $\frac{\pi}{2}$ to the left, which gives:



This is the vertical flip of the sin x graph, which is $-\sin x$.

With Calculator Portion Name: Key

•Note that once you finish this portion of the test and turn it in, you CANNOT return to it!

change?

add Law of cosines

5. (5 points) Solve the equation

$$\log_{10}(x+2) + \log_{10}(x-1) = 1$$

(a) algebraically ~~and~~ or
(b) graphically.

In both (a) and (b) you should explain/show your work. You should get the same answer(s) for both.

(a) $\log_{10}(x+2)(x-1) = 1 \Rightarrow 10^1 = (x+2)(x-1)$

$10 = x^2 + x - 2$

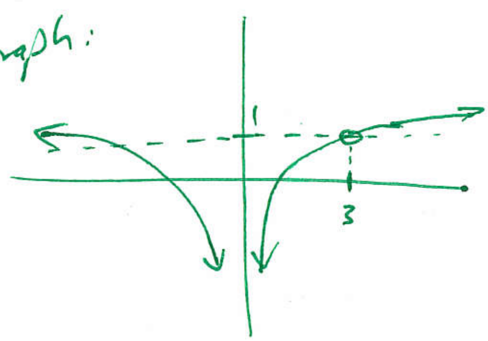
$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

So $x = -4$ or $\boxed{x = 3.}$

for $x = -4$, though, $\log(-4+2) = \log(-2)$ which is undefined, so $x = -4$ is not a solution

(b) Graph:



-1 if don't check -4

6. (5 points) In 2009, there were approximately 18 million Twitter users. In 2011, there were 117 million Twitter users.

4 (a) Use the equation for modeling exponential growth to estimate the number of Twitter users there were in ~~(i) 2008 and (ii)~~ 2016.

1 (b) The *actual* number of Twitter users in ~~2008 was 6 million, and in~~ 2016 it was 319 million. How close to the actual values ~~were~~ was your ~~estimates~~ estimate from part (a)? If ~~they were~~ it was close, briefly explain why ~~you think they were close~~, and If ~~they were~~ *not* ~~close~~, briefly explain why you think the ~~estimates were~~ estimate was not correct.

$A = Pe^{rt}$ Let $t = 0$ be 2009

then

$18 = Pe^{r9}$ and $117 = Pe^{r11}$

$\frac{18}{e^{r9}} = P$ $\frac{117}{e^{r11}} = P$

$\Rightarrow \frac{18}{e^{r9}} = \frac{117}{e^{r11}} \Rightarrow \frac{e^{r11}}{e^{r9}} = \frac{117}{18} \Rightarrow e^{2r} = \frac{117}{18}$

$2r = \ln\left(\frac{117}{18}\right)$

$r = \frac{1}{2}\ln\left(\frac{117}{18}\right) = 0.936$

8 $\Rightarrow P = \frac{18}{e^{(0.936)9}} = 0.003952$

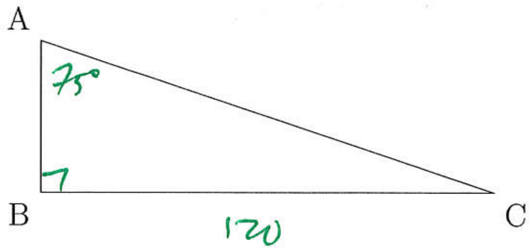
$\boxed{\cancel{(i)\ 2008 \Rightarrow A = 0.003952e^{0.936(8)}} = 7.06 \text{ million}}$

$(ii)\ 2016 \Rightarrow A = 0.00395e^{0.936(16)} = 12{,}612$ million

(b) Our estimate ~~for 2008 was very close, but for~~ 2016 it was <u>way</u> off. This is probably because Twitter growth was initially exponential (first few years) but the leveled off so the model does not fit for later yea

7. (5 points) Consider the triangle below, where angle $A = 75°$, angle B is a right angle, and side $\overline{BC} = 120$. Find the values of angle C, and sides $\overline{AB}$ and $\overline{AC}$.

Round to nearest 0.1.



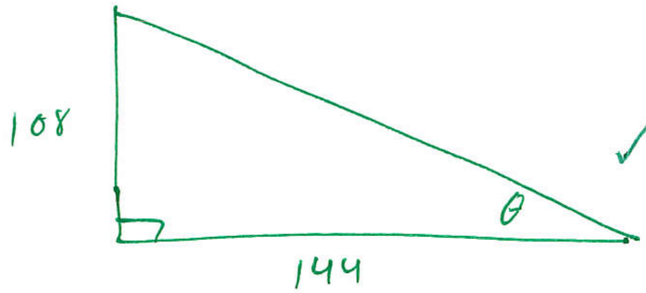
$\boxed{\angle C = 15^\circ}$ $(180 - 75^\circ - 90)$

$\cancel{\overline{AB} =}$ $\tan 75^\circ = \dfrac{120}{\overline{AB}} \Rightarrow \overline{AB} = \dfrac{120}{\tan(75)} = \boxed{32.2} = \overline{AB}$

$\overline{AC}^2 = 32.2^2 + 120^2 \Rightarrow \overline{AC} = \sqrt{15436.8} = \boxed{124.2} = \overline{AC}$

8. (5 points) A 108-foot tree casts a shadow that is 144 feet long. What is the angle of elevation of the sun?

↳ in degrees. Round to nearest 0.01



0.5

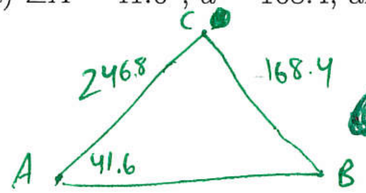
$$\tan \theta = \frac{108}{144}$$

0.5

$$\theta = \arctan\left(\frac{108}{144}\right) = \boxed{36.87^\circ}$$

9. (5 points) For each of the given triangles ABC, determine if there is one, multiple, or no ways to solve the triangle. Then solve the triangle(s) (if possible) by finding the remaining sides and angles. Round answers to the nearest 0.1, and explain your reasoning/show your work where appropriate.

(a) $\angle A = 41.6°$, $a = 168.4$, and $b = 246.8$



?

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin 41.6}{168.4} = \frac{\sin B}{246.8}$$

$$\Rightarrow B = \sin^{-1}\left(\frac{\sin 41.6 \cdot 246.8}{168.4}\right)$$

$\boxed{B = 76.7}$

$\angle C = 180 - 76.7 - 41.6$ or $B = 180 - 76.67$

$\boxed{\angle C = 61.7}$ $\boxed{B = 103.3}$

2 ways to solve!

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$c^2 = 246.8^2 + 168.4^2 - 2(246.8)(168.4)\cos(61.7)$$

$$c^2 = 49861.5 \Rightarrow \boxed{c = 223.3}$$

Other option

To find other triangle use this for angle

3.5

(b) $\angle A = 43.7°$, $a = 112.9$, and $b = 209.3$

$$\frac{\sin 43.7}{112.9} = \frac{\sin B}{209.3}$$

1.5

$$B = \arcsin\left(\frac{(\sin 43.7)(209.3)}{112.9}\right) = \arcsin(1.28)$$

undefined b/c 1.28 is greater than 1.

So there is no ~~way~~ such triangle!

10. (5 points) On a day when the sun travels directly overhead at noon, a 5-foot tall woman casts a shadow of length

$$D(t) = 5\left|\cot \frac{\pi}{12} t\right|$$

where D is measured in feet and t is the number of hours that have passed since 6 AM.

(a) (i) How long is the shadow at 7 AM? (ii) How long is the shadow at 2 PM?

~~(b) At what time(s) of the day is the woman's shadow exactly 3 feet long?~~

(c) At what time(s) of the day is the woman's shadow the same length as her height? Explain how to find the answer to this question *without* using a calculator (though you can use a calculator to check your work).

(d) What happens to the woman's shadow as ~~t~~ the day gets closer to 6 pm? Explain why the function D gives this result.

✓ (a) $D(1) = 5\left|\cot \frac{\pi}{12}\right| = \frac{5}{\left|\tan\left(\frac{\pi}{12}\right)\right|} = 18.66$ ft.

✓ $D(8) = 5\left|\cot \frac{8\pi}{12}\right| = 5\left|\cot \frac{2\pi}{3}\right| = 5\left|\frac{\left(-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right| = \frac{5}{\sqrt{3}} = \boxed{2}$ or

~~(b) $3 = 5\left|\cot \frac{\pi}{12} t\right|$~~

~~$\pm\frac{3}{5} = \frac{1}{\tan\left(\frac{\pi}{12}t\right)}$~~

~~$\tan\left(\frac{\pi}{12}t\right) = \pm\frac{5}{3}$~~

~~$t = \frac{12}{\pi}\tan^{-1}\left(\pm\frac{5}{3}\right)$~~ $\Rightarrow t = 3.9$ for $t = \frac{5}{3}$

$t = -3.9$ for $t = \frac{-5}{3}$

or $12 - 3.9 = 8.1$

✓ (c) $5 = 5\left|\cot \frac{\pi}{12} t\right|$

✓ $1 = \frac{1}{\left|\tan\left(\frac{\pi}{12}t\right)\right|}$

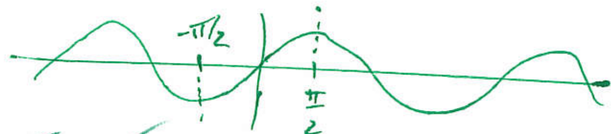
$\tan\left(\frac{\pi}{12}t\right) = \pm 1$ $\Rightarrow \frac{\pi}{12}t = \pm\frac{\pi}{4} \Rightarrow t = \pm 3$

$\boxed{\text{so at 9 AM and 3 PM}}$

✓ (d) As $t \to 12$, get close to $\cot\pi$, which is undefined b/c $\sin(\pi) = 0 \Rightarrow$ shadow gets longer and longer, to ∞.

11. (5 points) Explain, in your own words, why the inverse sine function ($\arcsin x$) is only defined for x-values in the domain $[\frac{-\pi}{2}, \frac{\pi}{2}]$. It is fine to use graphs or the unit circle to support your argument (in fact, it is recommended). and has range

sin x =



0.5 ✓ ✓ 0.5

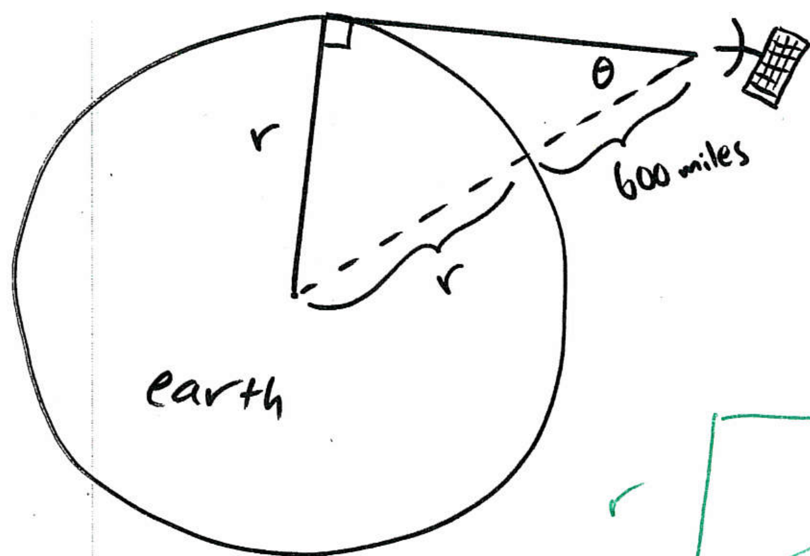
Sin x is not one-to-one, so we must restrict its domain to only $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ ✓

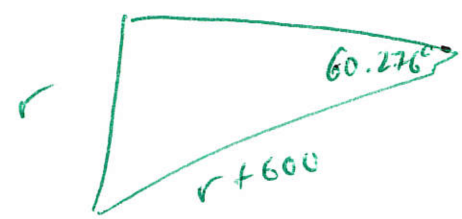
to get it to be 1-1 and have an inverse function. The inverse function will then have a domain of $[-1, 1]$ and range of $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. ✓

arcsin x

with range of $[-1, 1]$

Extra Credit: (2 points) From a satellite 600 miles above the earth, it is observed that the angle θ formed by the vertical and line of sight to the horizon is 60.276°. Explain how to use this information to find the radius of the earth. You do not actually have to find the radius, just explain how you would use math to do it.





$$\boxed{\sin(60.276°) = \frac{r}{r+600} \quad \text{and solve for } r}$$

$$0.868424(r+600) = r$$

$$(0.868424)(600) = 0.131576r$$

$$r = \frac{(0.868424)(600)}{0.131576}$$

$$r = 3960.1 \text{ miles}$$